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This article is the winner of the schools category of the Plus new writers award 2008. Students were asked to write about the life and work of a mathematician of their choice. "But the best problem I ever found, I found in my local public library." Andrew Wiles. Image © C. J. Mozzochi, Princeton N.J. There is a problem that not even the collective mathematical genius of almost 400 years could solve. When the ten-year-old Andrew Wiles read about it in his local Cambridge library, he dreamt of solving the problem that had haunted so many great mathematicians. Little did he or the rest of the world know that he would succeed... "Here was a problem, that I, a ten-year-old, could understand and I knew from that moment that I would never let it go. I had to solve it." Pierre de Fermat The story of the problem that would seal Wiles' place in history begins in 1637 when Pierre de Fermat made a deceptively simple conjecture. He stated that if n is any whole number greater than 2, then there are no three whole numbers a, b and c other than zero that satisfy the equation \\ \\ $a^n + b^n = c^n.$ \\ \\ (Note that if $n = 2$, then whole number solutions do exist, for example $a=3$, $b=4$ and $c=5$.) Fermat claimed to have proved this statement but that the "margin [was] too narrow to contain" it. It is the seeming simplicity of the problem, coupled with Fermat's claim to have proved it, which has captured the hearts of so many mathematicians. "Then when I reached college, I realized that many people had thought about the problem during the 18th and 19th centuries and so I studied those methods." Leonhard Euler Andrew Wiles was born in Cambridge, England on April 11 1953. At the age of ten he began to attempt to prove Fermat's last theorem using textbook methods. He then moved on to looking at the work of others who had attempted to prove the conjecture. Fermat himself had proved that for n=4 the equation had no solution, and Euler then extended Fermat's method to n=3. The problem was that to prove the general form of the conjecture, it does not help to prove individual cases; infinity minus something is still infinity. Wiles had to try a different approach in order to solve the problem. "However impenetrable it seems, if you don't try it, then you can never do it." A family of elliptic curves. Animation courtesy Aleksandar T. Lipkovski. Wiles earned a bachelors degree from Oxford University in 1974 and a PhD from Cambridge in 1980. It was while at Cambridge that he worked with John Coates on the arithmetic of elliptic curves. Elliptic curves are confusingly not much like an ellipse or a curve! They are defined by points in the plane whose co-ordinates x and y satisfy an equation of the form \\ \\ $y^2 = x^3 + \mu x + \lambda,$ \\ \\ where μ and λ are constants, and they are usually doughnut-shaped. When Wiles began studying elliptic curves they were an area of mathematics unrelated to Fermat's last theorem. But this was soon to change. Since the 1950s the Taniyama-Shimura conjecture had stated that every elliptic curve can be matched to a modular form — a mathematical object that is symmetrical in an infinite number of ways. Then in the summer of 1986 Ken Ribet, building on work of Gerhard Frey, established a link between Fermat's last theorem, elliptic curves and the Taniyama-Shimura conjecture. By showing a link between these three vastly different areas Ribet had changed the course of Wiles' life forever. "I was electrified. I knew that moment the course of my life was changing." What Ribet had managed to show, and what Frey had intuited, was that if Fermat's last theorem were false, that is if there were three non-zero whole numbers a, b and c, and a whole number n greater than 2 so that $$a^n + b^n = c^n,$$ then this would have very special consequences for the elliptic curve $$y^2 = x(x-a^n)(x-b^n),$$ which is known as the Frey curve: this curve would be unrelated to a modular form. If such an elliptic curve existed, then the Taniyama-Shimura conjecture would be false. Looking at this from a different perspective we can see that if the Taniyama-Shimura conjecture could be proved to be true, then the curve could not exist, hence Fermat's last theorem must be true. So to prove Fermat's last theorem, Wiles had to prove the Taniyama-Shimura conjecture. "You can't really focus yourself for years unless you have undivided concentration, which too many spectators would have destroyed" Proving the Taniyama-Shimura conjecture was an enormous task, one that many mathematicians considered impossible. Wiles decided that the only way he could prove it would be to work in secret at his Princeton home. He still performed his lecturing duties at the university but no longer attended conferences or told anyone what he was working on. This led many to believe he had finished as a mathematician; simply run out of ideas. After six years working alone, Wiles felt he had almost proved the conjecture. But he needed help from a friend called Nick Katz to examine one part of the proof. No problems were found and the moment to announce the proof came later that year at the Isaac Newton Institute in Cambridge. There it was that in June 1993 Andrew Wiles announced his historic proof of Fermat's Last Theorem. "It was so indescribably beautiful; it was so simple and elegant." Andrew Wiles Unfortunately for Wiles this was not the end of the story: his proof was found to contain a flaw. The flaw in the proof cannot be simply explained; however without rectifying the error, Fermat's last theorem would remain unsolved. After a year of effort, partly in collaboration with Richard Taylor, Wiles managed to fix the problem by merging two approaches. Both of the approaches were on their own inadequate, but together they were perfect. So it came to be that after 358 years and 7 years of one man's undivided attention that Fermat's last theorem was finally solved. "I think I'll stop here."About the author Neil Pieprzak is currently in year 12 studying for his A levels at Notre Dame Sixth Form in Sheffield. His interest in this particular problem was sparked by reading the book Fermat's last theorem by Simon Singh, which gives a great insight into the history of the theorem for those who want to know more. Neil hopes to study maths at university in 2009, where he is looking forward to tackling some problems of his own. In his spare time he enjoys watching football and has a season ticket for Sheffield Wednesday Football Club. Plus would like to thank the London Mathematical Society and the Maths, Stats and Operational Research Network, as well as the journal Nature for their kind support of this competition. This article now forms part of our coverage of the cutting-edge research done at the Isaac Newton Institute for Mathematical Sciences (INI) in Cambridge. The INI is an international research centre and our neighbour here on the University of Cambridge's maths campus. It attracts leading mathematical scientists from all over the world, and is open to all. Visit www.newton.ac.uk to find out more. Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. On June 23, 1993, the mathematician Andrew Wiles gave the last of three lectures detailing his solution to Fermat's last theorem, a problem that had remained unsolved for three and a half centuries. Wiles' announcement caused a sensation, both within the mathematical community and in the media. Beyond providing a satisfying resolution to a long-standing problem, Wiles' work marks an important moment in the establishment of a bridge between two important, but seemingly very different, areas of mathematics. History demonstrates that many of the greatest breakthroughs in math involve making connections between seemingly disparate branches of the subject. These bridges allow mathematicians, like the two of us, to transport problems from one branch to another and gain access to new tools, techniques and insights. What is Fermat's last theorem? Fermat's last theorem is similar to the Pythagorean theorem, which states that the sides of any right triangle give a solution to the equation x2 + y2 = z2 . The Pythagorean theorem, named for the Ancient Greek philosopher Pythagorus, is a fundamental result in Euclidean geometry that relates the lengths of the sides of a right triangle. AmericanXplorer13 via Wikimedia Commons, CC BY-SA 3.0 Every differently sized triangle gives a different solution, and in fact there are infinitely many solutions where all three of x, y and z are whole numbers – the smallest example is x=3, y=4 and z=5. Fermat's last theorem is about what happens if the exponent changes to something greater than 2. Are there whole-number solutions to x3 + y3 = z3 ? What if the exponent is 10, or 50, or 30 million? Or, most generally, what about any positive number bigger than 2? The French mathematician Pierre de Fermat. Rolland Lefebvre via Wikimedia Commons Around the year 1637, Pierre de Fermat claimed that the answer was no, there are no three positive whole numbers that are a solution to xn + yn = zn for any n bigger than 2. The French mathematician scribbled this claim into the margins of his copy of a math textbook from ancient Greece, declaring that he had a marvelous proof that the margin was "too narrow to contain." Fermat's purported proof was never found, and his "last theorem" from the margins, published posthumously by his son, went on to plague mathematicians for centuries. Searching for a solution For the next 356 years, no one could find Fermat's missing proof, but no one could prove him wrong either – not even Homer Simpson. The theorem quickly gained a reputation for being incredibly difficult or even impossible to prove, with thousands of incorrect proofs put forward. The theorem even earned a spot in the Guinness World Records as the "most difficult math problem." That is not to say that there was no progress. Fermat himself had proved it for n=3 and n=4. Many other mathematicians, including the trailblazer Sophie Germain, contributed proofs for individual values of n, inspired by Fermat's methods. But knowing Fermat's last theorem is true for certain numbers isn't enough for mathematicians – we need to know it's true for infinitely many of them. Mathematicians wanted a proof that would work for all numbers bigger than 2 at once, but for centuries it seemed as though no such proof could be found. However, toward the end of the 20th century, a growing body of work suggested Fermat's last theorem should be true. At the heart of this work was something called the modularity conjecture, also known as the Taniyama-Shimura conjecture. A bridge between two worlds A graph of an elliptic curve. Googolplexian1221, CC BY-SA 4.0, via Wikimedia Commons The modularity conjecture proposed a connection between two seemingly unrelated mathematical objects: elliptic curves and modular forms. Elliptic curves are neither ellipses nor curves. They are doughnut-shaped spaces of solutions to cubic equations, like y2 = x3 – 3x + 1. A modular form is a kind of function which takes in certain complex numbers – numbers with two parts: a real part and an imaginary part – and outputs another complex number. What makes these functions special is that they are highly symmetrical, meaning there are lots of conditions on what they can look like. The symmetries of a modular form can be seen in how it transforms a disc. Linas Vepstas, CC BY-SA 3.0, via Wikimedia Commons There is no reason to expect that those two concepts are related, but that is what the modularity conjecture implied. Finally, a proof The modularity conjecture doesn't appear to say anything about equations like xn + yn = zn . But work by mathematicians in the 1980s showed a link between these new ideas and Fermat's old theorem. First, in 1985, Gerhard Frey realized that if Fermat was wrong and there could be a solution to xn + yn = zn for some n bigger than 2, that solution would produce a peculiar elliptic curve. Then Kenneth Ribet showed in 1986 that such a curve could not exist in a universe where the modularity conjecture was also true. Their work implied that if mathematicians could prove the modularity conjecture, then Fermat's last theorem had to be true. For many mathematicians, including Andrew Wiles, working on the modularity conjecture became a path to proving Fermat's last theorem. Wiles worked for seven years, mostly in secret, trying to prove this difficult conjecture. By 1993, he was close to having a proof of a special case of the modularity conjecture – which was all he needed to prove Fermat's last theorem. He presented his work in a series of lectures at the Isaac Newton Institute in June 1993. Though subsequent peer review found a gap in Wiles' proof, Wiles and his former student Richard Taylor worked for another year to fill in that gap and cement Fermat's last theorem as a mathematical truth. Lasting consequences The impacts of Fermat's last theorem and its solution continue to reverberate through the world of mathematics. In 2001, a group of researchers, including Taylor, gave a full proof of the modularity conjecture in a series of papers that were inspired by Wiles' work. This completed bridge between elliptic curves and modular forms has been – and will continue to be – foundational to understanding mathematics, even beyond Fermat's last theorem. Wiles' work is cited as beginning "a new era in number theory" and is central to important pieces of modern math, including a widely used encryption technique and a huge research effort known as the Langlands Program that aims to build a bridge between two fundamental areas of mathematics: algebraic number theory and harmonic analysis. Although Wiles worked mostly in isolation, he ultimately needed help from his peers to identify and fill in the gap in his original proof. Increasingly, mathematics today is a collaborative endeavor, as witnessed by what it took to finish proving the modularity conjecture. The problems are large and complex and often require a variety of expertise. Andrew Wiles on winning the Abel Prize, a high honor in mathematics, in 2016 for his work on Fermat's last theorem. So, finally, did Fermat really have a proof of his last theorem, as he claimed? Knowing what mathematicians know now, many of us today don't believe he did. Although Fermat was brilliant, he was sometimes wrong. Mathematicians can accept that he believed he had a proof, but it's unlikely that his proof would stand up to modern scrutiny. "I think I'll stop here." This is how, on 23rd of June 1993, Andrew Wiles ended his series of lectures at the Isaac Newton Institute in Cambridge. The applause, so witnesses report, was thunderous: Wiles had just delivered a proof of a result that had haunted mathematicians for over 350 years: Fermat's last theorem. The theorem is linked to maths that is over 2000 years old. Most people have heard of Pythagoras' theorem: if a, b and c are the sides of a right-angled triangle, with c the side opposite the right angle, then $$a^2+b^2=c^2.$$ There are infinitely many triples a, b and c of natural numbers which satisfy this equation. Examples are $(3,4,5)$, $(5,12,13)$ and $(6,8,10).$ They are called Pythagorean triples. Fermat's last theorem looks at similar equations but with different exponents. Are there triples of natural numbers that satisfy $$a^3+b^3=c^3,$$ or $$a^4+b^4=c^4,$$ or $$a^5+b^5=c^5?$$ More generally, can we find triples of natural numbers that satisfy $$a^n+b^n=c^n,$$ where n is any natural number? The 17th century mathematician Pierre de Fermat convinced himself that such triples don't exist for any natural number $n>2$. In 1637 he wrote into the margin of his maths textbook that he had found a "marvellous proof" for this fact, which the margin was too narrow to contain. That tantalising scribble was to taunt mathematicians for a long time. Watch Andrew Wiles talk about what it feels like to do maths. When Wiles announced his proof at the Newton Institute he had spent seven years working on the problem in secret, avoiding the attention he would have attracted had he admitted to what he was doing. In fact he had been working on a more general problem, called the Taniyama-Shimura conjecture, whose solution implied Fermat's last theorem, but which was considered pretty inaccessible by most mathematicians at the time. "But Although Wiles hadn't published anything on this topic, everyone knew if anyone was to prove Fermat's last theorem, he had a good chance," recalls Tom Körner, a mathematician at the University of Cambridge. "I had interviewed Wiles for a fellowship at Trinity Hall, Cambridge, just before he moved to Princeton. Everyone said he was brilliant." Before Wiles announced his proof he had already lectured for two days, as part of a research programme at the Newton Institute. "Rumours were flying around that something important would happen on the third day," says Körner. "I asked his student what was happening and he said 'I can't tell you'. I asked him 'Will I regret not being there on the last day?' and he said 'Yes'." Wiles' lecture lived up to expectations: "The atmosphere was electric," says Körner. "After he finished, there was a storm of applause and then the few people in the audience who knew the material asked questions, and then said the result looked good." And it is not all about the bottom line. "Wiles' proof has had a greater impact than that of other results, like the four colour theorem, because of the mathematics that was created to prove the theorem," says Körner. As it turned out, Wiles' proof did contain a flaw which took another year and help from a former student, Richard Taylor, to fix. But in the end the 357-year-old-problem yielded to the sophisticated techniques developed to crack it. Further reading You can find out more in the Plus magazine article Fermat's last theorem and Andrew Wiles. 1995 publication in mathematics Sir Andrew John Wiles Wiles's proof of Fermat's Last Theorem is a proof by British mathematician Sir Andrew Wiles of a special case of the modularity theorem for elliptic curves. Together with Ribet's theorem, it provides a proof for Fermat's Last Theorem. Both Fermat's Last Theorem and the modularity theorem were believed to be impossible to prove using previous knowledge by almost all living mathematicians at the time.[1]:203–205,223,226 Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".[2] However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community. The corrected proof was published in 1995.[3] Wiles's proof uses many techniques from algebraic geometry and number theory and has many ramifications in these branches of mathematics. It also uses standard constructions of modern algebraic geometry such as the category of schemes, significant number theoretic ideas from Iwasawa theory, and other 20th-century techniques which were not available to Fermat. The proof's method of identification of a deformation ring with a Hecke algebra (now referred to as an R=T theorem) to prove modularity lifting theorems has been an influential development in algebraic number theory. Together, the two papers which contain the proof are 129 pages long[4][5] and consumed more than seven years of Wiles's research time. John Coates described the proof as one of the highest achievements of number theory, and John Conway called it "the proof of the [20th] century."[6] Wiles's path to proving Fermat's Last Theorem, by way of proving the modularity theorem for the special case of semistable elliptic curves, established powerful modularity lifting techniques and opened up entire new approaches to numerous other problems. For proving Fermat's Last Theorem, he was knighted, and received other honours such as the 2016 Abel Prize. When announcing that Wiles had won the Abel Prize, the Norwegian Academy of Science and Letters described his achievement as a "stunning proof".[3] Main article: Fermat's Last Theorem Fermat's Last Theorem, formulated in 1637, states that no three positive integers a, b, and c can satisfy the equation a n + b n = c n {\displaystyle a^{n}+b^{n}=c^{n}} if n is an integer greater than two (n > 2). Over time, this simple assertion became one of the most famous unproved claims in mathematics. Between its publication and Andrew Wiles's eventual solution more than 350 years later, many mathematicians and amateurs attempted to prove this statement, either for all values of n > 2, or for specific cases. It spurred the development of entire new areas within number theory. Proofs were eventually found for all values of n up to around 4 million, first by hand, and later by computer. However, no general proof was found that would be valid for all possible values of n, nor even a hint how such a proof could be undertaken. Main article: Modularity theorem Separately from anything related to Fermat's Last Theorem, in the 1950s and 1960s Japanese mathematician Goro Shimura, drawing on ideas posed by Yutaka Taniyama, conjectured that a connection might exist between elliptic curves and modular forms. These were mathematical objects with no known connection between them. Taniyama and Shimura posed the question whether, unknown to mathematicians, the two kinds of object were actually identical mathematical objects, just seen in different ways. They conjectured that every rational elliptic curve is also modular. This became known as the Taniyama–Shimura conjecture. In the West, this conjecture became well known through a 1967 paper by André Weil, who gave conceptual evidence for it; thus, it is sometimes called the Taniyama–Shimura–Weil conjecture. By around 1980, much evidence had been accumulated to form conjectures about elliptic curves, and many papers had been written which examined the consequences if the conjecture were true, but the actual conjecture itself was unproven and generally considered inaccessible—meaning that mathematicians believed a proof of the conjecture was probably impossible using current knowledge. For decades, the conjecture remained an important but unsolved problem in mathematics. Around 50 years after first being proposed, the conjecture was finally proven and renamed the modularity theorem, largely as a result of Andrew Wiles's work described below. On yet another separate branch of development, in the late 1960s, Yves Hellegouarch came up with the idea of associating hypothetical solutions (a, b, c) of Fermat's equation with a completely different mathematical object: an elliptic curve.[7] The curve consists of all points in the plane whose coordinates (x, y) satisfy the relation y 2 = x (x − a n) (x + b n) . {\displaystyle y^{2}=x(x-a^{n})(x+b^{n}).} Such an elliptic curve would enjoy very special properties due to the appearance of high powers of integers in its equation and the fact that an + bn = cn would be an nth power as well. In 1982–1985, Gerhard Frey called attention to the unusual properties of this same curve, now called a Frey curve. He showed that it was likely that the curve could link Fermat and Taniyama, since any counterexample to Fermat's Last Theorem would probably also imply that an elliptic curve existed that was not modular. Frey showed that there were good reasons to believe that any set of numbers (a, b, c, n) capable of disproving Fermat's Last Theorem could also probably be used to disprove the Taniyama–Shimura–Weil conjecture. Therefore, if the Taniyama–Shimura–Weil conjecture were true, no set of numbers capable of disproving Fermat could exist, so Fermat's Last Theorem would have to be true as well. The conjecture says that each elliptic curve with rational coefficients can be constructed in an entirely different way, not by giving its equation but by using modular functions to parametrise coordinates x and y of the points on it. Thus, according to the conjecture, any elliptic curve over Q would have to be a modular elliptic curve, yet if a solution to Fermat's equation with non-zero a, b, c and n greater than 2 existed, the corresponding curve would not be modular, resulting in a contradiction. If the link identified by Frey could be proven, then in turn, it would mean that a disproof of Fermat's Last Theorem would disprove the Taniyama–Shimura–Weil conjecture, or by contraposition, a proof of the latter would prove the former as well.[8] Main article: Ribet's theorem To complete this link, it was necessary to show that Frey's intuition was correct: that a Frey curve, if it existed, could not be modular. In 1985, Jean-Pierre Serre provided a partial proof that a Frey curve could not be modular. Serre did not provide a complete proof of his proposal; the missing part (which Serre had noticed early on[9]:1) became known as the epsilon conjecture (sometimes written ε-conjecture; now known as Ribet's theorem). Serre's main interest was in an even more ambitious conjecture, Serre's conjecture on modular Galois representations, which would imply the Taniyama–Shimura–Weil conjecture. However his partial proof came close to confirming the link between Fermat and Taniyama. In the summer of 1986, Ken Ribet succeeded in proving the epsilon conjecture, now known as Ribet's theorem. His article was published in 1990. In doing so, Ribet finally proved the link between the two theorems by confirming, as Frey had suggested, that a proof of the Taniyama–Shimura–Weil conjecture for the kinds of elliptic curves Frey had identified, together with Ribet's theorem, would also prove Fermat's Last Theorem. In mathematical terms, Ribet's theorem showed that if the Galois representation associated with an elliptic curve has certain properties (which Frey's curve has), then that curve cannot be modular, in the sense that there cannot exist a modular form which gives rise to the same Galois representation.[10] Following the developments related to the Frey curve, and its link to both Fermat and Taniyama, a proof of Fermat's Last Theorem would follow from a proof of the Taniyama–Shimura–Weil conjecture—or at least a proof of the conjecture for the kinds of elliptic curves that included Frey's equation (known as semistable elliptic curves). From Ribet's Theorem and the Frey curve, any 4 numbers able to be used to disprove Fermat's Last Theorem could also be used to make a semistable elliptic curve ("Frey's curve") that could never be modular; But if the Taniyama–Shimura–Weil conjecture were also true for semistable elliptic curves, then by definition every Frey's curve that existed must be modular. The contradiction could have only one answer: if Ribet's theorem and the Taniyama–Shimura–Weil conjecture for semistable curves were both true, then it would mean there could not be any solutions to Fermat's equation—because then there would be no Frey curves at all, meaning no contradictions would exist. This would finally prove Fermat's Last Theorem. However, despite the progress made by Serre and Ribet, this approach to Fermat was widely considered unusable as well, since almost all mathematicians saw the Taniyama–Shimura–Weil conjecture itself as completely inaccessible to proof with current knowledge.[1]:203–205,223,226 For example, Wiles's ex-supervisor John Coates stated that it seemed "impossible to actually prove",[1]:226 and Ken Ribet considered himself "one of the vast majority of people who believed [it] was completely inaccessible".[1]:223 Hearing of Ribet's 1986 proof of the epsilon conjecture, English mathematician Andrew Wiles, who had studied elliptic curves and had a childhood fascination with Fermat, decided to begin working in secret towards a proof of the Taniyama–Shimura–Weil conjecture, since it was now professionally justifiable,[11] as well as because of the enticing goal of proving such a long-standing problem. Ribet later commented that "Andrew Wiles was probably one of the few people on earth who had the audacity to dream that you can actually go and prove [it]."[1]:223 Wiles initially presented his proof in 1993. It was finally accepted as correct, and published, in 1995, following the correction of a subtle error in one part of his original paper. His work was extended to a full proof of the modularity theorem over the following six years by others, who built on Wiles's work. During 21–23 June 1993, Wiles announced and presented his proof of the Taniyama–Shimura conjecture for semistable elliptic curves, and hence of Fermat's Last Theorem, over the course of three lectures delivered at the Isaac Newton Institute for Mathematical Sciences in Cambridge, England.[2] There was a relatively large amount of press coverage afterwards.[12] After the announcement, Nick Katz was appointed as one of the referees to review Wiles's manuscript. In the course of his review, he asked Wiles a series of clarifying questions that led Wiles to recognise that the proof contained a gap. There was an error in one critical portion of the proof which gave a bound for the order of a particular group: the Euler system used to extend Kolyvagin and Flach's method was incomplete. The error would not have rendered his work worthless—each part of Wiles's work was highly significant and innovative by itself, as were the many developments and techniques he had created in the course of his work, and only one part was affected.[1]:289,296–297 Without this part proved, however, there was no actual proof of Fermat's Last Theorem. Wiles spent almost a year trying to repair his proof, initially by himself and then in collaboration with his former student Richard Taylor, without success.[13][14][15] By the end of 1993, rumours had spread that under scrutiny, Wiles's proof had failed, but how seriously was not known. Mathematicians were beginning to pressure Wiles to disclose his work whether or not complete, so that the wider community could explore and use whatever he had managed to accomplish. Instead of being fixed, the problem, which had originally seemed minor, now seemed very significant, far more serious, and less easy to resolve.[16] Wiles states that on the morning of 19 September 1994, he was on the verge of giving up and was almost resigned to accepting that he had failed, and to publishing his work so that others could build on it and find the error. He states that he was having a final look to try to understand the fundamental reasons why his approach could not be made to work, when he had a sudden insight that the specific reason why the Kolyvagin–Flach approach would not work directly also meant that his original attempt using Iwasawa theory could be made to work if he strengthened it using experience gained from the Kolyvagin–Flach approach since then. Each was inadequate by itself, but fixing one approach with tools from the other would resolve the issue and produce a class number formula (CNF) valid for all cases that were not already proven by his refereed paper:[13][17] I was sitting at my desk examining the Kolyvagin–Flach method. It wasn't that I believed I could make it work, but I thought that at least I could explain why it didn't work. Suddenly I had this incredible revelation. I realised that, the Kolyvagin–Flach method wasn't working, but it was all I needed to make my original Iwasawa theory work from three years earlier. So out of the ashes of Kolyvagin–Flach seemed to rise the true answer to the problem. It was so indescribably beautiful; it was so simple and so elegant. I couldn't understand how I'd missed it and I just stared at it in disbelief for twenty minutes. Then during the day I walked around the department, and I'd keep coming back to my desk looking to see if it was still there. It was still there. I couldn't contain myself, I was so excited. It was the most important moment of my working life. Nothing I ever do again will mean as much.—Andrew Wiles, quoted by Simon Singh[18] On 6 October Wiles asked three colleagues (including Gerd Faltings) to review his new proof,[19] and on 24 October 1994 Wiles submitted two manuscripts, "Modular elliptic curves and Fermat's Last Theorem"[4] and "Ring theoretic properties of certain Hecke algebras",[5] the second of which Wiles had written with Taylor and proved that certain conditions were met which were needed to justify the corrected step in the main paper. The two papers were vetted and finally published as the entirety of the May 1995 issue of the Annals of Mathematics. The new proof was widely analysed and became accepted as likely correct in its major components.[6][10][11] These papers established the modularity theorem for semistable elliptic curves, the last step in proving Fermat's Last Theorem, 358 years after it was conjectured. Fermat claimed to "... have discovered a truly marvelous proof of this, which this margin is too narrow to contain".[20][21] Wiles's proof is very complex, and incorporates the work of so many other specialists that it was suggested in 1994 that only a small number of people were capable of fully understanding at that time all the details of what he had done.[2][22] The complexity of Wiles's proof motivated a 10-day conference at Boston University; the resulting book of conference proceedings aimed to make the full range of required topics accessible to graduate students in number theory.[9] As noted above, Wiles proved the Taniyama–Shimura–Weil conjecture for the special case of semistable elliptic curves, rather than for all elliptic curves. Over the following years, Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor (sometimes abbreviated as "BCDT") carried the work further, ultimately proving the Taniyama–Shimura–Weil conjecture for all elliptic curves in a 2001 paper.[23] Now proven, the conjecture became known as the modularity theorem. In 2005, Dutch computer scientist Jan Bergstra posed the problem of formalizing Wiles's proof in such a way that it could be verified by computer.[24] This section needs attention from an expert in mathematics. The specific problem is: Newly added section: review required for technical accuracy. WikiProject Mathematics may be able to help recruit an expert. (June 2017) Wiles proved the modularity theorem for semistable elliptic curves, from which Fermat's last theorem follows using proof by contradiction. In this proof method, one assumes the opposite of what is to be proved, and shows if that were true, it would create a contradiction. The contradiction shows that the assumption (that the conclusion is wrong) must have been incorrect, requiring the conclusion to hold. The proof falls roughly in two parts: In the first part, Wiles proves a general result about "lifts", known as the "modularity lifting theorem". This first part allows him to prove results about elliptic curves by converting them to problems about Galois representations of elliptic curves. He then uses this result to prove that all semistable curves are modular, by proving that the Galois representations of these curves are modular. Outline proof Comment Part 1: setting up the proof 1 We start by assuming (for the sake of contradiction) that Fermat's Last Theorem is incorrect. That would mean there is at least one non-zero solution (a, b, c, n) (with all numbers rational and n > 2 and prime) to an + bn = cn. 2 Ribet's theorem (using Frey and Serre's work) shows that using the solution (a, b, c, n), we can create a semistable elliptic Frey curve (which we will call E) which is never modular. If we can prove that all such elliptic curves will be modular (meaning that they match a modular form), then we have our contradiction and have proved our assumption (that such a set of numbers exists) was wrong. If the assumption is wrong, that means no such numbers exist, which proves Fermat's Last Theorem is correct. Part 2: the modularity lifting theorem 3 Galois representations of elliptic curves ρ(E, p) for any prime p > 3 have been studied by many mathematicians. Wiles aims first of all to prove a result about these representations, that he will use later: that if a semistable elliptic curve E has a Galois representation ρ(E, p) that is modular, the elliptic curve itself must be modular. Proving this is helpful in two ways: it makes counting and matching easier, and, significantly, to prove the representation is modular, we would only have to prove it for one single prime number p, and we can do this using any prime that makes our work easy – it does not matter which prime we use. This is the most difficult part of the problem – technically it means proving that if the Galois representation ρ(E, p) is a modular form, so are all the other related Galois representations ρ(E, p∞) for all powers of p.[3] This is the so-called "modular lifting problem", and Wiles approached it using deformations. Any elliptic curve (or a representation of an elliptic curve) can be categorized as either reducible or irreducible. The proof will be slightly different depending whether or not the elliptic curve's representation is reducible. Comparing elliptic curves and modular forms directly is difficult; past efforts to count and match elliptic curves and modular forms had all failed. However, since elliptic curves can be represented within Galois theory, Wiles realized that working with the representations of elliptic curves instead of the curves themselves would make counting and matching them to modular forms far easier. From this point on, the proof primarily aims to prove: (1) if the geometric Galois representation of a semistable elliptic curve is modular, so is the curve itself; and (2) the geometric Galois representations of all semistable elliptic curves are modular. Together, these allow us to work with representations of curves rather than directly with elliptic curves themselves. Our original goal will have been transformed into proving the modularity of geometric Galois representations of semistable elliptic curves, instead. Wiles described this realization as a "key breakthrough". A Galois representation of an elliptic curve is G → GL(Zp). To show that a geometric Galois representation of an elliptic curve is a modular form, we need to find a normalized eigenform whose eigenvalues (which are also its Fourier series coefficients) satisfy a congruence relationship for all but a finite number of primes. 4 Wiles's initial strategy is to count and match using proof by induction and a class number formula ("CNF"): an approach in which, once the hypothesis is proved for one elliptic curve, it can automatically be extended to be proven for all subsequent elliptic curves. It was in this area that Wiles found difficulties, first with horizontal Iwasawa theory and later with his extension of Kolyvagin–Flach. Wiles's work extending Kolyvagin–Flach was mainly related to making Kolyvagin–Flach strong enough to prove the full CNF he would use. It later turned out that neither of these approaches by itself could produce a CNF able to cover all types of semistable elliptic curves, and the final piece of his proof in 1995 was to realize that he could succeed by strengthening Iwasawa theory with the techniques from Kolyvagin–Flach. 5 At this point, the proof has shown a key point about Galois representations: If the geometric Galois representation ρ(E, p) of a semistable elliptic curve E is irreducible and modular (for some prime number p > 2), then subject to some technical conditions, E is modular. This is Wiles's lifting theorem (or modularity lifting theorem), a major and revolutionary accomplishment at the time. Crucially, this result does not just show that modular irreducible representations imply modular curves. It also means we can prove a representation is modular by using any prime number > 2 that we find easiest to use (because proving it for just one prime > 2 proves it for all primes > 2). So we can try to prove all of our elliptic curves are modular by using one prime number as p - but if we do not succeed in proving this for all elliptic curves, perhaps we can prove the rest by choosing different prime numbers as 'p' for the difficult cases. The proof must cover the Galois representations of all semistable elliptic curves E, but for each individual curve, we only need to prove it is modular using one prime number p.) Part 3: Proving that all semistable elliptic curves are modular 6 With the lifting theorem proved, we return to the original problem. We will categorize all semistable elliptic curves based on the reducibility of their Galois representations, and use the powerful lifting theorem on the results. From above, it does not matter which prime is chosen for the representations. We can use any one prime number that is easiest. 3 is the smallest prime number more than 2, and some work has already been done on representations of elliptic curves using ρ(E, 3), so choosing 3 as our prime number is a helpful starting point. Wiles found that it was easier to prove the representation was modular by choosing a prime p = 3 in the cases where the representation ρ(E, 3) is irreducible, but the proof when ρ(E, 3) is reducible was easier to prove by choosing p = 5. So, the proof splits in two at this point. The proof's use of both p = 3 and p = 5 below, is the so-called "3/5 switch" referred to in some descriptions of the proof, which Wiles noticed in a paper of Mazur's in 1993, although the trick itself dates back to the 19th century. The switch between p = 3 and p = 5 has since opened a significant area of study in its own right (see Serre's modularity conjecture). 7 If the Galois representation ρ(E, 3) (i.e., using p = 3) is irreducible, then it was known from around 1980 that its Galois representation is also always modular. Wiles uses his modularity lifting theorem to make short work of this case: If the representation ρ(E, 3) is irreducible, then we know the representation is also modular (Langlands and Tunnell), but... ... if the representation is both irreducible and modular then E itself is modular (modularity lifting theorem). Langlands and Tunnell proved this in two papers in the early 1980s. The proof is based on the fact that ρ(E, 3) has the same symmetry group as the general quartic equation in one variable, which was one of the few general classes of diophantine equation known at that time to be modular. This existing result for p = 3 is crucial to Wiles's approach and is one reason for initially using p = 3. 8 So we now consider what happens if ρ(E, 3) is reducible. Wiles found that when the representation of an elliptic curve using p = 3 is reducible, it was easier to work with p = 5 and use his new lifting theorem to prove that ρ(E, 5) will always be modular, than to try and prove directly that ρ(E, 3) itself is modular (remembering that we only need to prove it for one prime). 5 is the next prime number after 3, and any prime number can be used, perhaps 5 will be an easier prime number to work with than 3? But it looks hopeless initially to prove that ρ(E, 5) is always modular, for much the same reason that the general quintic equation cannot be solved by radicals. So Wiles has to find a way around this. 8.1 If ρ(E, 3) and ρ(E, 5) are both reducible, Wiles proved directly that ρ(E, 5) must be modular. 8.2 The last case is if ρ(E, 3) is reducible and ρ(E, 5) is irreducible. Wiles showed that in this case, one could always find another semistable elliptic curve F such that the representation ρ(F, 3) is irreducible and also the representations ρ(E, 5) and ρ(F, 5) are isomorphic (they have identical structures). - The first of these properties shows that F must be modular (Langlands and Tunnell again: all irreducible representations with p = 3 are modular). - If F is modular then we know ρ(F, 5) must be modular as well. - But because the representations of E and F with p = 5 have exactly the same structure, and we know that ρ(F, 5) is modular, ρ(E, 5) must be modular as well. 8.3 Therefore, if ρ(E, 3) is reducible, we have proved that ρ(E, 5) will always be modular. But if ρ(E, 5) is modular, then the modularity lifting theorem shows that E itself is modular. This step shows the real power of the modularity lifting theorem. Results 9 We have now proved that whether or not ρ(E, 3) is irreducible, E (which could be any semistable elliptic curve) will always be modular. This means that all semistable elliptic curves must be modular. This proves: (a) The Taniyama–Shimura–Weil conjecture for semistable elliptic curves; and also (b) Because there cannot be a contradiction, it also proves that the kinds of elliptic curves described by Frey cannot actually exist. Therefore no solutions to Fermat's equation can exist either, so Fermat's Last Theorem is also true. We have our proof by contradiction, because we have proven that if Fermat's Last Theorem is incorrect, we could create a semistable elliptic curve that cannot be modular (Ribet's Theorem) and must be modular (Wiles). As it cannot be both, the only answer is that no such curve exists. Wiles opted to attempt to match elliptic curves to a countable set of modular forms. He found that this direct approach was not working, so he transformed the problem by instead matching the Galois representations of the elliptic curves to modular forms. Wiles denotes this matching (or mapping) that, more specifically, is a ring homomorphism: R n → T n . {\displaystyle R_{n}\rightarrow \mathbf {T} _{n}.} R {\displaystyle R} is a deformation ring and T {\displaystyle \mathbf {T} } is a Hecke ring. Wiles had the insight that in many cases this ring homomorphism could be a ring isomorphism (Conjecture 2.16 in Chapter 2, §3 of the 1995 paper[4]). He realised that the map between R {\displaystyle R} and T {\displaystyle \mathbf {T} } is an isomorphism if and only if two abelian groups occurring in the theory are finite and have the same cardinality. This is sometimes referred to as the "numerical criterion". Given this result, Fermat's Last Theorem is reduced to the statement that two groups have the same order. Much of the text of the proof leads into topics and theorems related to ring theory and commutation theory. Wiles's goal was to verify that the map R → T {\displaystyle R\rightarrow \mathbf {T} } is an isomorphism and ultimately that R = T {\displaystyle R=\mathbf {T} } . In treating deformations, Wiles defined four cases, with the flat deformation case requiring more effort to prove and treated in a separate article in the same volume entitled "Ring-theoretic properties of certain Hecke algebras". Gerd Faltings, in his bulletin, gives the following commutative diagram (p. 745): or ultimately that R = T {\displaystyle R=\mathbf {T} } , indicating a complete intersection. Since Wiles could not show that R = T {\displaystyle R=\mathbf {T} } directly, he did so through Z 3 , F 3 {\displaystyle \mathbf {Z} _{3},\mathbf {F} _{3}} and T / m {\displaystyle \mathbf {T} /{\mathfrak {m}}} via lifts. In order to perform this matching, Wiles had to create a class number formula (CNF). He first attempted to use horizontal Iwasawa theory but that part of his work had an unresolved issue such that he could not create a CNF. At the end of the summer of 1991, he learned about an Euler system recently developed by Victor Kolyvagin and Matthias Flach that seemed "tailor made" for the inductive part of his proof, which could be used to create a CNF, and so Wiles set his Iwasawa work aside and began working to extend Kolyvagin and Flach's work instead, in order to create the CNF his proof would require.[25] By the spring of 1993, his work had covered all but a few families of elliptic curves, and in early 1993, Wiles was confident enough of his nearing success to let one trusted colleague into his secret. Since his work relied extensively on using the Kolyvagin–Flach approach, which was new to mathematics and to Wiles, and which he had also extended, in January 1993 he asked his Princeton colleague, Nick Katz, to help him review his work for subtle errors. Their conclusion at the time was that the techniques Wiles used seemed to work correctly.[1]:261–265[26] Wiles's use of Kolyvagin–Flach would later be found to be the point of failure in the original proof submission, and he eventually had to revert to Iwasawa theory and a collaboration with Richard Taylor to fix it. In May 1993, while reading a paper by Mazur, Wiles had the insight that the 3/5 switch would resolve the final issues and would then cover all elliptic curves. Given an elliptic curve E {\displaystyle E} over the field Q {\displaystyle \mathbb {Q} } of rational numbers, for every prime power ℓ n {\displaystyle \ell ^{n}} , there exists a homomorphism from the absolute Galois group Gal (Q ¯ / Q) {\displaystyle \operatorname {Gal} ({\bar {\mathbb {Q} }}/\mathbb {Q})} to GL 2 (Z / ℓ n Z) , {\displaystyle \operatorname {GL} _{2}(\mathbb {Z} /\ell ^{n}\mathbb {Z}),} the group of invertible 2 by 2 matrices whose entries are integers modulo ℓ n {\displaystyle \ell ^{n}} . This is because E (Q ¯) {\displaystyle E({\bar {\mathbb {Q} }})} , the points of E {\displaystyle E} over Q ¯ {\displaystyle {\bar {\mathbb {Q} }}} , form an abelian group on which Gal (Q ¯ / Q) {\displaystyle \operatorname {Gal} ({\bar {\mathbb {Q} }}/\mathbb {Q})} acts; the subgroup of elements x {\displaystyle x} such that ℓ n x = 0 {\displaystyle \ell ^{n}x=0} is just (Z / ℓ n Z) 2 {\displaystyle (\mathbb {Z} /\ell ^{n}\mathbb {Z})^{2}} , and an automorphism of this group is a matrix of the type described. Less obvious is that given a modular form of a certain special type, a Hecke eigenform with eigenvalues in Q {\displaystyle \mathbb {Q} } , one also gets a homomorphism Gal (Q ¯ / Q) → GL 2 (Z / ℓ n Z) . {\displaystyle \operatorname {Gal} ({\bar {\mathbb {Q} }}/\mathbb {Q})\rightarrow \operatorname {GL} _{2}(\mathbb {Z} /\ell ^{n}\mathbb {Z}).} This goes back to Eichler and Shimura. The idea is that the Galois group acts first on the modular curve on which the modular form is defined, thence on the Jacobian variety of the curve, and finally on the points of ℓ n {\displaystyle \ell ^{n}} power order on that Jacobian. The resulting representation is not usually 2-dimensional, but the Hecke operators cut out a 2-dimensional piece. It is easy to demonstrate that these representations come from some elliptic curve but the converse is the difficult part to prove. Instead of trying to go directly from the elliptic curve to the modular form, one can first pass to the mod ℓ n {\displaystyle {\bmod {\ell ^{n}}}} representation for some ℓ {\displaystyle \ell } and n {\displaystyle n} , and from that to the modular form. In the case where ℓ = 3 {\displaystyle \ell =3} and n = 1 {\displaystyle n=1} , results of the Langlands–Tunnell theorem show that the mod 3 {\displaystyle {\bmod {3}}} representation of any elliptic curve over Q {\displaystyle \mathbb {Q} } comes from a modular form. The basic strategy is to use induction on n {\displaystyle n} to show that this is true for ℓ = 3 {\displaystyle \ell =3} and any n {\displaystyle n} , that ultimately there is a single modular form that works for all n. To do this, one uses a counting argument, comparing the number of ways in which one can lift a mod ℓ n {\displaystyle \ell ^{n}} Galois representation to one mod ℓ n + 1 {\displaystyle \ell ^{n+1}} and the number of ways in which one can lift a mod ℓ n {\displaystyle \ell ^{n}} modular form. An essential point is to impose a sufficient set of conditions on the Galois representation; otherwise, there will be too many lifts and most will not be modular. These conditions should be satisfied for the representations coming from modular forms and those coming from elliptic curves. If the original (m o d 3) {\displaystyle (\mathrm {mod} \,3)} representation has an image which is too small, one runs into trouble with the lifting argument, and in this case, there is a final trick which has since been studied in greater generality in the subsequent work on the Serre modularity conjecture. The idea involves the interplay between the (m o d 3) {\displaystyle (\mathrm {mod} \,3)} and (m o d 5) {\displaystyle (\mathrm {mod} \,5)} representations. In particular, if the mod-5 Galois representation ρ ¯ E , 5 {\displaystyle {\overline {\rho }}_{E,5}} associated to an semistable elliptic curve E over Q is irreducible, then there is another semistable elliptic curve E' over Q such that its associated mod-5 Galois representation ρ ¯ E ′ , 5 {\displaystyle {\overline {\rho }}_{E',5}} is isomorphic to ρ ¯ E , 5 {\displaystyle {\overline {\rho }}_{E,5}} and such that its associated mod-3 Galois representation ρ ¯ E , 3 {\displaystyle {\overline {\rho }}_{E,3}} is irreducible (and therefore modular by Langlands–Tunnell).[27] In his 108-page article published in 1995, Wiles divides the subject matter up into the following chapters (preceded here by page numbers): Introduction 443 Chapter 1 455 1. Deformations of Galois representations 472 2. Some computations of cohomology groups 475 3. Some results on subgroups of GL2(k) Chapter 2 479 1. The Gorenstein property 489 2. Congruences between Hecke rings 503 3. The main conjectures Chapter 3 517 Estimates for the Selmer group Chapter 4 525 1. The ordinary CM case 533 2. Calculation of η Chapter 5 541 Application to elliptic curves Appendix 545 Gorenstein rings and local complete intersections Gerd Faltings subsequently provided some simplifications to the 1995 proof, primarily in switching from geometric constructions to rather simpler algebraic ones.[19][28] The book of the Cornell conference also contained simplifications to the original proof.[9] Wiles's paper is more than 100 pages long and often uses the specialised symbols and notations of group theory, algebraic geometry, commutative algebra, and Galois theory. The mathematicians who helped to lay the groundwork for Wiles often created new specialised concepts and technical jargon. Among the introductory presentations are an email which Ribet sent in 1993;[29][30] Hesselink's quick review of top-level issues, which gives just the elementary algebra and avoids abstract algebra;[24] or Daney's web page, which provides a set of his own notes and lists the current books available on the subject. Weston attempts to provide a handy map of some of the relationships between the subjects.[31] F. Q. Gouvêa's 1994 article "A Marvelous Proof", which reviews some of the required topics, won a Lester R. Ford award from the Mathematical Association of America.[32][33] Faltings' 5-page technical bulletin on the matter is a quick and technical review of the proof for the non-specialist.[34] For those in search of a commercially available book to guide them, he recommended that those familiar with abstract algebra read Hellegouarch, then read the Cornell book,[9] which is claimed to be accessible to "a graduate student in number theory". The Cornell book does not cover the entirety of the Wiles proof.[12] Abstract algebra p-adic number Semistable curves

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The title of one edition of the PBS television series NOVA discusses Andrew Wiles's effort to prove Fermat's Last Theorem that broadcast on BBC Horizon and UTV/Documentary as Fermat's Last Theorem (Adobe Flash) (subscription required) Wiles, Ribet, Shimura–Taniyama–Weil and Fermat's Last Theorem Are mathematicians finally satisfied with Andrew Wiles's proof of Fermat's Last Theorem? Why has this theorem been so difficult to prove?, Scientific American, 21 October 1999 The Man Who Solved the World's Hardest Math Problem on YouTube Overview of Wiles proof, accessible to non-experts, by Henri Darmon Very short summary of the proof by Charles Daney 140 page students work-through of the proof, with exercises, by Nigel Boston Retrieved from " "I think I'll stop here." This is how, on 23rd June 1993, Andrew Wiles ended his series of lectures at the Isaac Newton Institute, our neighbour here at the Centre for Mathematical Sciences. The applause, so witnesses report, was thunderous. Wiles had just delivered a proof that had eluded mathematicians for over 350 years: Fermat's Last Theorem. An infamous scribble The theorem concerns equations of the form xn+yn=zn where n is a natural number. The question is whether there are triples of non-zero natural numbers x,y,z, that satisfy such an equation. For n=2 the answer is yes. There are in fact infinitely many such triples, known as Pythagorean triples, because the numbers involved also satisfy Pythagoras' theorem for right-angled triangles. "This was the first time that I had seen a human story attached to a mathematical problem. Not just the story of one person, but people talking to each other over a period of centuries." Jack Thorne The 17th century mathematician Pierre de Fermat convinced himself that when the exponent n is greater than 2, however, there are no integer solutions to the equation. In 1637 he wrote into the margin of his maths textbook that he had found a "marvellous proof" for this fact, which the margin was too narrow to contain. That tantalising scribble was to taunt mathematicians for a long time. Andrew Wiles was one of them. "I first found out about Fermat's Last theorem from the cover of a book by E.T. Bell when I was about ten years old," says Wiles, who earned his PhD here at Cambridge in 1980, and is now Regius Professor in Mathematics at the University of Oxford. "I was captured by the romantic history of [the problem], so I spent some of my teenage years and even [some time] in college trying to solve it. But then when I became a professional mathematician I realised that this was not something you should be working on because it probably wouldn't generate any results." In the mid-1980s, however, work by the mathematicians Gerhard Frey, Jean-Pierre Serre and Ken Ribet provided a new way of attacking Fermat's Last Theorem. It showed that if you could prove another result, known as the modularity conjecture (also known as the Taniyama-Shimura-Weil conjecture) then you'd have automatically proved Fermat's Last Theorem too. "I was sceptical when the first announcement came out, but when Ribet proved that connection I was completely hooked and I dropped everything and started working on Fermat straight away," says Wiles. Unusually for a mathematician, he decided to work on the problem alone and in secret, for a period of seven years. "Very few people want to work on a problem for that long. To really commit yourself to a problem takes a certain kind of personality. I did initially [talk about it] a tiny bit, but then I realised that it got so much unwanted attention when you said you were working on it, you wouldn't be left in peace. So I felt it was wiser to do it in private." An electric atmosphere The result which Wiles finally proved was the modularity conjecture, in a setting that was general enough to imply that Fermat's Last Theorem was also true. He announced his proof at the Isaac Newton Institute on June 23, 1993. The announcement came at the end of a series of three lectures and nobody really knew that this was what Wiles had had in store. "Rumours started to get around," says Professor Tom Körner of the Department of Pure Mathematics and Mathematical Statistics at Cambridge, who had the privilege of witnessing the lecture. "I do not know if people knew or just speculated, so I asked one of Andrew's students whether I would regret missing the lecture, and he said yes. The atmosphere was electric." "At the end of [the lecture] Andrew wrote up the statement of Fermat's Last Theorem, and indicated that what he had done, he felt, had proved it. There was tremendous applause and then the experts got up and asked questions which indicated that, although details of the proof remained to be thoroughly checked, it was a very plausible way of attacking the problem. It was also a new way of attacking the problem, so that whether it succeeded or not, it had added a substantial amount to mathematics." "On the one hand I was very excited to present [the result], but on the other there is always a tension the first time [you share the work]," says Wiles when recalling the announcement. "You have been thinking about this [for a long time], a lot of it on your own, so you [hope that you] haven't done anything stupid. I think people wanted to see the details, but they could see that this was a completely new approach and that it was going to prove something - whether it had all the details of the final claim remained to be seen." The desire to see the details proved justified: it turned out the proof as it stood contained a hole, which it took Wiles, together with the mathematician Richard Taylor (also a Cambridge alumnus who had been Wiles' former PhD student at Princeton), nearly a year to fix. But then finally, in 1994, the centuries old problem that was so tantalisingly inspired by a note scribbled in the margin of a book was finally solved. The future of mathematics You might think that, when an old problem is finally solved, a door closes on the area of mathematics involved. But this is rarely the case, as a solved problem usually opens up a range of unsolved ones. Wiles says that Fermat's Last Theorem has sparked two periods of intense progress in the past: one in the 19th century when the foundations for Wiles' areas of mathematics were laid in attempts to prove Fermat's Last Theorem, and one in the 1980s, which finally lead to the proof. The proof itself, Wiles says, has helped to ring in a new era. "It opened another door, this time on problems of modularity. And these problems of modularity are themselves just one more door opening on this great perspective of what is called the Langlands programme — that's the future of mathematics." It is difficult to explain the Langlands programme even to an expert, suffice to say that it consists of a web of far reaching conjectures made by Robert Langlands in the 1960s that draws extremely surprising connections between different fields of mathematics. Proving all these conjectures is seen by many as the single biggest project of modern mathematics. The Langlands programme attracts some of the brightest minds in mathematics. Among them is Professor Jack Thorne of the Department of Pure Mathematics and Mathematical Statistics here at Cambridge. Thorne was six years old when Wiles announced the proof of Fermat's Last Theorem, and became interested in the result while doing his mathematics A levels. "I found it quite exciting because doing A Level maths you learn how to do certain kinds of calculations, for example how to balance two balls on a rod and things like that," he says. "But this was the first time that I had seen a human story attached to a mathematical problem. Not just the story of one person, but people talking to each other over a period of centuries." Despite his young age, Thorne is already a leading expert in his field. He has won a number of prizes, including the prestigious New Horizons in Mathematics Prize, and became the youngest living fellow of the Royal Society when he was elected in 2020. Thorne works on the Langlands programme, in particular on the connection it provides between number theory on the one hand and an area of maths that comes from generalisations of objects called modular forms on the other. "They are two worlds [for] which, a priori, it is not clear they should be connected, but [which] talk to each other in ways that are very mysterious and very striking," he explains. "It's really like there's a hidden telephone line." The Langlands programme provides new tools for attacking problems in number theory. Thorne has used these tools to consider equations similar to the one of Fermat, but slightly more general: rather than requiring all the coefficients in the equation to be integers, you can ask yourself what happens if the coefficients come from larger number fields, for example fields containing more awkward irrational numbers such as the square root of 2. For some such classes of equations the theory generalised beautifully, says Thorne, but much work is still needed to push the field further. Wiles agrees that extending our theory of arithmetic to encompass more general number fields, using the tools provided by the Langlands programme, is one of the most important challenges of the future. So while Wiles' proof settled a problem that is so easy to state that even a high school student can understand it, it has opened the door on a deep new area of mathematics which will see exciting developments in the next decade or so, in which mathematicians like Thorne are likely to play a leading role. A key moment That moment 30 years ago was clearly a turning point in Wiles' career. He is one of the few mathematicians who is well-known outside of mathematics, and was recognised with a knighthood in 2000. Within mathematics he has received a wealth of honours and awards, including the prestigious Abel Prize in 2016. It's like we've been on this grand, often bewildering scavenger hunt to figure out the secrets of the universe. And what's increasingly clear is that the universe, in all its cheeky complexity, prefers to talk in the language of mathematics. It's as though we've been handed a cosmic crossword puzzle where every clue is written in numbers and equations. Fermat's last theorem is one of the most famous and challenging problems in the history of mathematics. It was first stated by Pierre de Fermat, a French lawyer and mathematician, in the margin of a copy of Arithmetica, a book by the ancient Greek mathematician Diophantus.What follows is a centuries-long wild goose chase. Fermat never did publish this mysterious proof, leaving generations of mathematicians to scratch their heads and wonder whether he was pulling the world's longest leg or had genuinely cracked one of math's toughest nuts. It's the kind of thing that makes you wonder if history's greatest mathematical minds weren't just having us on for a laugh.Source: Trinity College LibraryHis son, Samuel, found the note after his father's death and published it in 1670.The theorem states that for any integer n greater than 2, there are no positive integers x, y, and z that satisfy the following equation,Image by the authorFor example, when n = 2, the equation is the Pythagorean theorem, which has infinitely many solutions, such as x = 3, y = 4, and z = 5. However, when n is larger than 2, Fermat claimed that there are no solutions at all.For more than three centuries, many mathematicians tried to prove or disprove Fermat's last theorem, but none succeeded. Some of them managed to prove the theorem for some special cases, such as n = 3, 4, 5, or 7, but the general case remained elusive. The theorem became one of the most famous unsolved problems in mathematics, and attracted the attention of both professional and amateur mathematicians. Many incorrect proofs were published and refuted over the years, and some prizes were offered for a valid proof.The first complete proof of Fermat's last theorem was given by Andrew Wiles, a British mathematician, in 1994. Wiles had been fascinated by the problem since he was 10 years old, and he spent seven years working on it in secret at Princeton University. His proof was based on a connection between the theorem and another conjecture in number theory, called the Taniyama-Shimura conjecture, which relates elliptic curves and modular forms. Wiles used a technique called modularity lifting to show that the Taniyama-Shimura conjecture implies Fermat's last theorem, and then he proved the conjecture for a large class of elliptic curves.Here's a simplified outline of his proof:Taniyama-Shimura-Weil Conjecture (Modularity Theorem): This conjecture, proposed in the 1950s and 1960s, suggested a deep link between two seemingly unrelated areas of mathematics: elliptic curves (complex equations that describe a certain set of points) and modular forms (certain complex functions with highly symmetric properties). The conjecture proposed that every elliptic curve is modular, meaning it can be associated with a modular form.Frey's Curve: In the 1980s, Gerhard Frey noted that if Fermat's Last Theorem were false (i.e., if there were a set of integers a,b,c,and n>2 that satisfied Fermat's equation), one could construct a very unusual elliptic curve (later known as Frey's curve). This curve had properties that seemed to contradict the Taniyama-Shimura-Weil conjecture.Ribet's Theorem: Building on Frey's idea, Ken Ribet proved in 1990 that if such a Frey curve existed, it would indeed contradict the Taniyama-Shimura-Weil conjecture. Ribet's theorem effectively linked Fermat's Last Theorem with the conjecture: if the conjecture were true, Fermat's Last Theorem would have to be true as well.Wiles' ProofAndrew Wiles, who had been fascinated by Fermat's Last Theorem since childhood, focused on proving the Taniyama-Shimura-Weil conjecture for a special class of elliptic curves that would include the hypothetical Frey curve. After years of intense work, he announced his proof in a series of lectures at Cambridge University in June 1993, and it was met with great excitement and applause by the mathematical community. However, in August 1993, a gap was discovered in his proof, which required a major revision. Wiles worked with his former student Richard Taylor to fix the gap, and they succeeded in September 1994. Their revised proof was published in the Annals of Mathematics in 1995, and it was widely accepted as correct. Wiles received many honors and awards for his achievement, including the Abel Prize, the highest prize in mathematics, in 2016. His proof of Fermat's last theorem is considered one of the most remarkable and beautiful results in the history of mathematics. It was a monumental piece of work that drew upon many areas of mathematics, including algebraic geometry, number theory, and the theory of elliptic curves."Mathematics... is a bit like discovering oil. But mathematics has one great advantage over oil, in that no one has yet found a way that you can keep using the same oil forever." — Andrew Wiles, in his 1997 AMS SpeechThank you so much for reading. If you liked this story don't forget to press that clap icon as many times as you want. If you like my works and want to support me then you can Buy me a coffee ☕. Keep following for more such stories! Read on the Plus website here: Words: Marianne Freiberger & Rachel Thomas "I think I'll stop here." This is how, on 23rd June 1993, Andrew Wiles ended his series of lectures at the Isaac Newton Institute. The applause, so witnesses report, was thunderous. Wiles had just delivered a proof that had eluded mathematicians for over 350 years: Fermat's Last Theorem. An infamous scribble The theorem concerns equations of the form: xn+yn=zn where n is a natural number. The question is whether there are triples of non-zero natural numbers x,y,z, that satisfy such an equation. For n=2 the answer is yes. There are in fact infinitely many such triples, known as Pythagorean triples, because the numbers involved also satisfy Pythagoras' theorem for right-angled triangles. 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"I found it quite exciting because doing A Level maths you learn how to do certain kinds of calculations, for example how to balance two balls on a rod and things like that," he says. "But this was the first time that I had seen a human story attached to a mathematical problem. Not just the story of one person, but people talking to each other over a period of centuries." Despite his young age, Thorne is already a leading expert in his field. He has won a number of prizes, including the prestigious New Horizons in Mathematics prize, and became the youngest living fellow of the Royal Society when he was elected in 2020. Thorne works on the Langlands programme, in particular on the connection it provides between number theory on the one hand and an area of maths that comes from generalisations of objects called modular forms on the other. 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Wiles told us, back in 2016, about some of the personal qualities a mathematician has to have – they have to be creative, and they have to be able to enjoy being stuck. And perseverance again appeared as a key thread in the story when we spoke to him for this article. Our final question was whether he would have kept on working on Fermat's Last Theorem even if he hadn't found a solution back in the early 90s. His answer was characteristic of his approach to mathematics: "I am not a person who gives up on a problem." , the free encyclopedia that anyone can edit. 110,331 active editors 7,014,564 articles in English HMS Neptune was a dreadnought battleship built for the Royal Navy in the first decade of the 20th century, the sole ship of her class. Laid down at HM Dockyard, Portsmouth, in January 1909, she was the first British battleship to be built with superfiring guns. Shortly after her completion in 1911, she carried out trials of an experimental fire-control director and then became the flagship of the Home Fleet. Neptune became a private ship in early 1914 and was assigned to the 1st Battle Squadron. The ship became part of the Grand Fleet when it was formed shortly after the beginning of the First World War in August 1914. Aside from participating in the Battle of Jutland in May 1916, and the inconclusive action of 19 August several months later, her service during the war generally consisted of routine patrols and training in the North Sea. Neptune was deemed obsolete after the war and was reduced to reserve before being sold for scrap in 1922 and subsequently broken up. (Full article...) Recently featured: Nominative determinism Donkey Kong Land History of education in Wales (1701–1870) Archive By email More featured articles About Cui Daozhi ... that Cui Daozhi (pictured) has been called the Chinese Sherlock Holmes? ... that "whatever the Glossa does not recognize, the court does not recognize"? ... that American football players Tommy Akingbesote and Kyonte Hamilton grew up in the same community, play in the same position, and were both selected in the seventh round of the 2025 NFL draft? ... that Mighty Doom features a cartoonish aesthetic that contrasts with the dark, gritty tone of the mainline Doom series? ... that sprinter Abdul Wahab Zahiri made his international debut in the same year that he competed at the Olympics? ... that the 14th-century builder of Gaza's Zofor Domri Mosque was buried in the mosque? ... that actor Ben Ahlers learned watchmaking from the Horological Society of New York for his role in The Gilded Age? ... that due to difficulty reading the biography Edison, one reviewer read it backwards? ... that Horace Niall served as magistrate, defence lawyer, executioner, and coroner – for the same group of men? 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Casani Richard Gerald Jordan Franco Testa Raymond Laflamme Gertrud Leutenegger Maria Voce Nominate an article June 28: Vidovdan in Serbia Ned Kelly 1880 – Police captured Australian bank robber and cultural icon Ned Kelly (pictured) after a gun battle in Glenrowan, Victoria. 1895 – The U.S. Court of Private Land Claims ruled that James Reavis's claim to 18,600 sq mi (48,000 km2) of land in present-day Arizona and New Mexico was "wholly fictitious and fraudulent". 1904 – In the worst maritime disaster involving a Danish merchant ship, SS Norge ran aground on Hasselwood Rock and sank in the North Atlantic, resulting in more than 635 deaths. 1950 – Korean War: South Korean forces began the Bodo League massacre, summarily executing tens of thousands of suspected North Korean sympathizers. 1969 – In response to a police raid at the Stonewall Inn in New York City, groups of gay and transgender people began demonstrations, a watershed event for the worldwide gay rights movement. Charles Cruft (b. 1852)Olga Sapphire (b. 1907)Meralda Warren (b. 1959)Aparna Rao (d. 2005) More anniversaries: June 27 June 28 June 29 Archive By email List of days of the year About Myosotis scorpioides, the water forget-me-not, is a herbaceous perennial flowering plant in the borage family, Boraginaceae. It is native to Europe and Asia, but is widely distributed elsewhere, including much of North America, as an introduced species and sometimes a noxious weed. It is an erect to ascending plant of up to 70 cm, bearing small (8-12 mm) flowers that become blue when fully open and have yellow centers. It is usually found in damp or wet habitats, such as bogs, ponds, streams, ditches, fen and rivers. This focus-stacked photograph shows a water forget-me-not growing in Niitvälja bog, Estonia. Photograph credit: Ivar Leidus Recently featured: Whitehead's trogon Atacamite Turban Head eagle Archive More featured pictures Community portal – The central hub for editors, with resources, links, tasks, and announcements. Village pump – Forum for discussions about Wikipedia itself, including policies and technical issues. Site news – Sources of news about Wikipedia and the broader Wikimedia movement. Teahouse – Ask basic questions about using or editing Wikipedia. Help desk – Ask questions about using or editing Wikipedia. Reference desk – Ask research questions about encyclopedic topics. Content portals – A unique way to navigate the encyclopedia. Wikipedia is written by volunteer editors and hosted by the Wikimedia Foundation, a non-profit organization that also hosts a range of other volunteer projects: CommonsFree media repository MediaWikiWiki software development Meta-WikiWikimedia project coordination WikibooksFree textbooks and manuals WikidataFree knowledge base WikinewsFree-content news WikiquoteCollection of quotations WikisourceFree-content library WikispeciesDirectory of species WikiversityFree learning tools WikivoyageFree travel guide WiktionaryDictionary and thesaurus This Wikipedia is written in English. Many other Wikipedias are available; some of the largest are listed below. 1,000,000+ articles العربية Deutsch Español فارسی Français Italiano Nederlands 日本語 Polski Português Русский Svenska Українська Tiếng Việt 中文 250,000+ articles Bahasa Indonesia Bahasa Melayu Bân-lâm-gú Български Català Čeština Dansk Eesti Ελληνικά Esperanto Euskara עברית Հայերեն Magyar Norsk bokmål Română Simple English Slovenčina Srpski Srpskohrvatski Suomi Türkçe O'zbekcha 50,000+ articles Asturianu Azərbaycanca বাংলা Bosanski کوردی Frysk Gaeilge Galego Hrvatski ქართული Kurdî Latviešu Lietuvių മലയാളം Македонски मराठी Norsk nynorsk ਪੰਜਾਬੀ Shqip Slovenščina தமிழ் ภาษาไทย اردو Retrieved from " 2Battleship formation of the Royal Navy For the German counterpart during World War I, see I Battle Squadron. 1st Battle SquadronThe 1st Battle Squadron at sea, April 1915Active1912–1945Country United KingdomBranch Royal NavyTypeSquadronSize8 x BattleshipsPart ofGrand FleetMilitary unit The 1st Battle Squadron was a naval squadron of the British Royal Navy consisting of battleships. The 1st Battle Squadron was initially part of the Royal Navy's Grand Fleet. After World War I the Grand Fleet was reverted to its original name, the Atlantic Fleet. The squadron changed composition often as ships were damaged, retired or transferred. As an element in the Grand Fleet, the Squadron participated in the Battle of Jutland.[1] On 5 August 1914, the squadron was constituted as follows:[2] HMS Marlborough HMS Collingwood HMS Colossus HMS Hercules HMS Neptune HMS St. Vincent HMS Superb HMS Vanguard Revenge and Hercules en route to Jutland with the sixth division. During the Battle of Jutland, the composition of the 1st Battle Squadron was as follows:[1] Sixth Division HMS Marlborough Flagship of Vice-Admiral Sir Cecil Burney; Captain G. P. Ross; HMS Revenge Captain E. B. Kiddle; HMS Hercules Captain L. Clinton-Baker; HMS Agincourt Captain H. M. Doughty; Fifth Division HMS Colossus Flagship of Rear Admiral E. F. A. Gaunt; Captain A. D. P. R. Pound; HMS Collingwood Captain J. C. Ley; HMS St. Vincent Captain W. W. Fisher; HMS Neptune Captain V. H. G. Bernard; HMS Revenge Following the Battle of Jutland, the 1st Battle Squadron was reorganized, with Colossus, Hercules, St. Vincent, Collingwood and Neptune all transferred to the 4th Battle Squadron. In January 1917, the squadron was constituted as follows:[3] HMS Marlborough HMS Agincourt HMS Benbow – joined July, 1916 HMS Canada HMS Emperor of India – joined July, 1916 HMS Revenge HMS Royal Oak – joined May, 1916 HMS Royal Sovereign – joined June, 1916 By 1918, Agincourt had been transferred to the 2nd Battle Squadron, and Resolution, Ramillies and Iron Duke had joined the squadron on completion.[4] For many years the squadron served in the Mediterranean as the main British battle force there. On 3 September 1939 the 1st Battle Squadron, serving in the Mediterranean Fleet, consisted of Barham, Warspite and Malaya, with headquarters at Alexandria, Egypt, under the command of Vice-Admiral Geoffrey Layton.[5] In December 1943 the Squadron was under the command of Vice Admiral Arthur Power. In January 1944 the Eastern Fleet was reinforced by HMS Queen Elizabeth, HMS Renown, HMS Valiant, HMS Illustrious, HMS Unicorn and seven destroyers. The Admiralty sent this force out to India under the title of the First Battle Squadron.[6] From November 1944, the squadron served in the British Pacific Fleet under the command of Vice-Admiral Henry Rawlings, who also served as Second-in-Command of the Fleet. It consisted of HMS King George V, HMS Howe, HMS Duke of York and HMS Anson at various times. Commanders were as follows:[7] Vice-Admiral Sir Stanley Colville (1912–14) Vice-Admiral Sir Lewis Bayly (June–December 1914) Admiral Sir Cecil Burney (1914–16) Vice-Admiral Sir Charles Madden (1916–19) Vice-Admiral Sir Sydney Fremantle (1919–21) Vice-Admiral Sir William Nicholson (1921–22) Vice-Admiral Sir Edwyn Alexander-Sinclair (1922–24) Rear-Admiral William Fisher (1924–25) Rear-Admiral Cecil Staveley (1925–26) Vice-Admiral Sir Michael Hodges (1926–27) Vice-Admiral Sir John Kelly (1927–29) Vice-Admiral Howard Kelly (1929–30) Vice-Admiral Sir William Fisher (1930–32) Vice-Admiral Sir Roger Backhouse (1932–34) Vice-Admiral Sir Charles Forbes (1934–36) Vice-Admiral Hugh Binney (1936–38) Rear-Admiral Ralph Leatham (1938–39) Vice-Admiral Geoffrey Layton (January–November 1939) Rear-Admiral Henry Pridham-Wippell (July–October 1940) Vice-Admiral John Tovey (October–December 1940) Rear-Admiral Bernard Rawlings (1940–41) Vice-Admiral Sir Henry Pridham-Wippell (1941–42) Vice-Admiral Sir Arthur Power (1943–44) Vice-Admiral Sir Bernard Rawlings (1944–45) Post holders included:[8] Rear-Admiral Charles E. Madden, 5 January 1912 – 10 November 1912 Rear-Admiral The Hon. Somerset A. Gough-Calthorpe, 10 December 1912 – 10 December 1913 Rear-Admiral Hugh Evan-Thomas, 10 December 1913 – 25 August 1915 Rear-Admiral Ernest Gaunt, 25 August 1915 – 12 June 1916 Rear-Admiral Alexander L. Duff, 12 June 1916 – 30 November 1916 Rear-Admiral Sir William C. M. Nicholson, 1 December 1916 – 20 March 1919 Rear-Admiral The Hon. Victor A. Stanley, 1 April 1919 – 1 April 1920 Rear-Admiral Henry M. Doughty, 24 March 1920 – 14 April 1921 Rear-Admiral Sir Rudolf W. Bentinck, 3 May 1921 – 3 May 1922 Rear-Admiral Arthur A. M. Duff, 3 May 1922 Rear-Admiral William A. H. Kelly, 3 May 1923 Rear-Admiral William H. D. Boyle, 3 May 1924 – 3 May 1924 Rear-Admiral William W. Fisher, 14 October 1924 – 7 September 1925 Rear-Admiral Cecil M. Staveley, 15 October 1925 – 1 October 1926 Rear-Admiral David T. Norris, 1 October 1926 Rear-Admiral Bernard St. G. Collard, 1 October 1927 Rear-Admiral William M. Kerr, 20 March 1928 – 5 April 1929 Rear-Admiral The Hon. Reginald A. R. P.-E.-E.-Drax, 12 April 1929 – 26 April 1930 Rear-Admiral Henry D. Pridham-Wippell, 8 May 1940 – 24 October, 1941 ^ a b Macintyre, Donald. Jutland Evans Brothers Ltd. 1957; ISBN 0-330-20142-5 ^ Dittmar, F.J & Colledge J.J., British Warships 1914–1919 Ian Allan, London. 1972; ISBN 0-7110-0380-7 ^ Dittmar, F.J & Colledge J.J., British Warships 1914–1919 Ian Allan, London. 1972; ISBN 0-7110-0380-7 pp20 ^ Dittmar, F.J & Colledge J.J., British Warships 1914–1919 Ian Allan, London. 1972; ISBN 0-7110-0380-7 pp24 ^ Orbat.com/Niehorster, Mediterranean Fleet, 3 September 1939, accessed May 2008 ^ Jackson, Ashley (2006). The British Empire and the Second World War. Continuum International Publishing Group. p. 301. ISBN 1-85285-417-0. ^ "Royal Navy Senior Appointments" (PDF). Archived from the original (PDF) on 11 July 2011. Retrieved 4 October 2014. ^ Harley, Simon; Lovell, Tony. "First Battle Squadron (Royal Navy) - The Dreadnought Project". www.dreadnoughtproject.org. Harley and Lovell, 27 December 2016. Retrieved 15 February 2018. First Battle Squadron at DreadnoughtProject.org Royal Navy History Composition of the Grand Fleet Retrieved from " 3 The following pages link to 1st Battle Squadron External tools (link count transclusion count sorted list) · See help page for transcluding these entries Showing 50 items. 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